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RESISTANCE OF TUBES TO COLLAPSE.[1]

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THE purpose of this paper is to describe some experiments made on collapsing small brass tubes by hydrostatic pressure, and to compare these experimental results with the empirical and the theoretical formulæ which have been proposed for the resistance of metal tubes to collapse.

The only extended and systematic experiments which have been published on the collapse of tubes, are the experiments of Sir William Fairbairn made neary fifty years ago at the suggestion of the Royal Society and the British Association. Fairbairn's results are given in his paper in the Philosophical Transactions for 1858. The empirical formula which he deduced from his experiments has since been in general use in some form by engineers in the practical calculations of the strength of boiler flues. Fairbairn in his paper gives the diameters, lengths, thicknesses, and collapsing pressures for twenty-one thin iron tubes, and for five thicker tubes. As no two of the thicker tubes were exactly alike in construction, the formula deduced was almost wholly based on the experiments with the thin tubes. These thin tubes were made of single sheets of tinned iron plate, bent into a cylindrical shape, riveted and soldered. All were 0.043 of an inch thick. Five diameters, 4″, 6″, 8″, 10″, and 12″, were used in the experiments, and the lengths ranged between 19 inches and 60 inches. Fairbairn's empirical formula is

$$p = 806{,}300\,\frac{t^{2.19}}{ld}$$

where p equals the external pressure in pounds per square inch, t equals the thickness of the walls in inches, l equals the diameter in feet, and d equals the diameter in inches.

This formula obviously holds only for limited lengths, for the collapsing pressure can not decrease indefinitely as the lengths in-

[1] A paper read before the American Physical Society in Chicago, April 23, 1905.

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crease. The numerical constant must also vary with the elastic properties of the material. The formulæ adopted by the British Board of Trade, Lloyd's Marine Underwriters, and the U. S. Inspectors of Steam Vessels, have no theoretical value, being merely working variations of Fairbairn's formula. Many writers have shown the insufficiency of the above formula and have studied the data and deduced new empirical formulæ. Such are the formulæ of Grashof[1] of G. H. Love[2] and W. C. Unwin.[3] Professor Unwin was an assistant in Fairbairn's experiments, and nearly thirty years later, gave a discussion of the same results, seeking to deduce a formula which should have some relation to the ordinary formulæ of applied mechanics. His formula was based on no new experiments and has not been accepted, its insufficiency having been shown by several writers.

Theoretical treatment of the problem has been presented within a few years by three Cambridge mathematicians, G. H. Bryan[4] A. B. Bassett[5] and A. E. H. Love.[6] The problem is one in the stability of an elastic system, and, as Love says, is one of the most difficult in the theory of elasticity. Mr. Bassett in his paper in the Phil. Mag., for September, 1892, "On the Difficulties of Constructing a Theory of the Collapse of Boiler-Flues," attempts, however, to show that while the solution proposed by Mr. Bryan and himself is imperfect and not rigorous, yet it is probable for practical purposes a sufficient approximation. Bryan, Bassett and Love, with slightly different methods and assumptions, came to substantially the same expression for the collapsing pressure of a tube of indefinite length. Love's statement is: "No flue however long can collapse unless the pressure exceed $2E/(1-\sigma^2)\, h^3/a^3$ and that when the pressure exceeds this limit any flue will collapse, if its length exceed a certain multiple of the mean proportional between the diameter and the thickness." In the above expression, E is Young's modulus, σ is Poisson's ratio, h is half the thickness of the wall, and a is the radius of the middle surface of the tube. No experimental

[1] Zeitschrift d. Vereins deutscher Ingenieure, vol. 3, p. 234.
[2] Todhunter and Pearson's History of Elasticity, vol. 2, p. 667.
[3] Proc. of Inst. C. E., 1876, vol. 46, p. 225.
[4] Proc. Camb. Phil. Soc., VI., 1888.
[5] Phil. Mag., vol. 34 (1892), p. 221.
[6] Theory of Elasticity, vol. II., p. 315.

tests of these theoretical deductions have been published so far as we know.

The experiments described here were started to test the empirical formula of Fairbairn, particularly for fixing a limit on the length to which that formula is applicable, a point altogether omitted by Fairbairn. The experiments soon showed the narrow limits and the inadequacy of Fairbairn's formula. Attention was then drawn to the theoretical solution of the three mathematicians. The experiments were made on seamless brass tubes of small diameters. The tubes were placed in the stout steel receiver of a Cailletet apparatus, and hydrostatic pressure was produced by a Cailletet pump. It was found that the impact which came with the collapse of the tube injured the hard rubber valves of the pump, and the

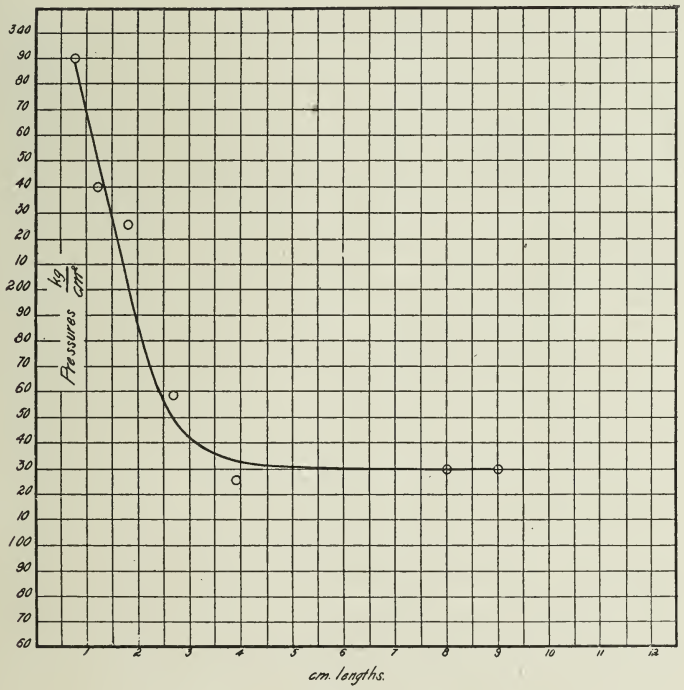

Fig. 1.

time required to get the valves working again after the collapse of each tube, made the experiments very tedious. So a valve was put in to cut off the pump from the receiver after a sufficient prelimi-

nary pressure was reached. The final pressure was then produced by a piston compressor, the piston being forced in by a screw. This auxiliary piston compressor was constructed specially for this work in the machine shop of the department. This arrangement had an added advantage that the pressure was increased gradually and the exact collapsing pressure could be more easily read. The pressures were read from a Bourdon gauge made by Shaeffer and Budenberg. The ends of the tube were closed by brass plugs soldered in, and the inside of the tube was connected with the atmosphere. The collapse came suddenly, and the hydrostatic pressure fell immediately to atmospheric pressure. As may be seen from the data given below, pressure up to 858 kilograms per square centimeter were used. The tubes in the experiments were commercial seamless brass tubes, purchased from the same jobber,

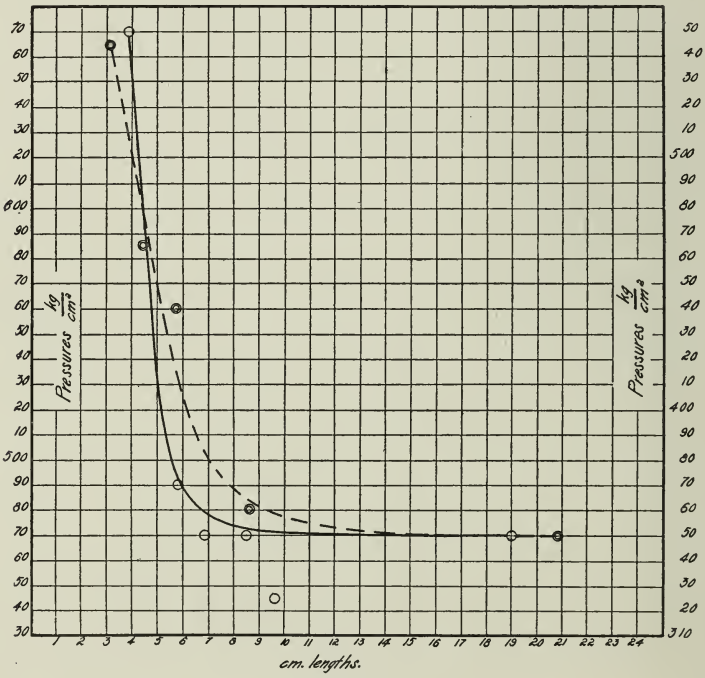

Fig. 2.

with the idea of obtaining uniform material, but unfortunately the brass in the different tubes was not the same. In any one size

there was, however, remarkable uniformity, as shown by the tests. A considerable number of tests were made on brazed tubes purchased in the market, but there was such a variation in the results in samples from the same piece of tubing, that nothing could be made of the tests. With larger tubes this might not be found the case. The lengths, diameters, thicknesses, and the collapsing pressures are given in the following table and curves. I am indebted to Mr. F. C. Brown for many of the observations.

Table of Collapsing Pressures for Seamless Brass Tubes.

Mean Diameter.	Thickness of Wall.	Length.	Collapsing Pressure.
.891 cm.	.041 cm.	8 mm.	290 kg./cm.^{2}
"	"	12 "	240 "
"	"	18 "	225 "
"	"	27 "	158 "
"	"	39 "	125 "
"	"	80 "	130 "
"	"	90 "	130 "
"	"	39 "	670 "
1.12 cm.	.08 cm.	58 "	490 "
"	"	69 "	470 "
"	"	85 "	470 "
"	"	97 "	450 "
"	"	190 "	470 "
1.83 cm.	.08 cm.	31 "	545 "
"	"	44 "	465 "
"	"	58 "	440 "
"	"	77 "	360 "
"	"	208 "	350 "
"	"	87 "	840 "
1.78 cm.	0.135 cm.	89 "	858 "
"	"	130 "	850 "
"	"	130 "	845 "
"	"	132 "	840 "
"	"	134 "	850 "
"	"	210 "	850 "

An inspection of these data and curves, Figs. 1 and 2, shows immediately that there is a minimum length for each tube beyond which the collapsing pressure is constant, and further, that this minimum length is quite definite. Again, we see that for lengths less than this critical minimum length, the collapsing pressures rise rapidly. As definitely as can be determined from these small tubes,

the collapsing pressure varies inversely as the length, for lengths less than the critical length. In this they follow Fairbairn's formula, and suggest that Fairbairn's tubes were all less than their critical lengths. An inspection of the woodcuts which Fairbairn gives for each of his experiments and comparing them with the shapes of the brass tubes which we have collapsed confirms this. Figs. 3 and 4 show shapes and sections of the collapsed tubes of the curve

Fig. 3.

for Fig. 1. Fairbairn's shapes are exactly those which we have gotten for lengths less than the critical length. In the quotation made above from Love, it is stated that this minimum length is a multiple of the mean proportional between the thickness and the diameter. It is to be presumed that for the same material, this multiple should be a constant. But the data on these small tubes were not found sufficient to determine this factor of the mean pro-

Fig. 4.

portional. The annealing of the tubes is probably not the same. In the case of one size tube upon which we have excellent data, viz., the tube with a mean diameter of 1.78 cm. and a thickness of 0.135 cm., we were fortunate in getting a value of Young's modulus for the brass. This was kindly obtained by Professor A. N. Talbot in the Laboratory of Testing Materials of the University of Illinois. Using this value, 11.5×10^{11} dynes per square centimeter (a high value for brass), and assuming corresponding value for Poisson's

ratio, we calculated the value of the collapsing pressure from Bryan's formula, and got a value of 10.4×10^{11}. The experimental value was 8.5×10^{11}, a number of the same order, and probably as good a confirmation as could be expected from a single experiment.

These experiments on small brass tubes have shown that formulæ of the Fairbairn type are inadequate, and that for tubes of sufficient length, a formula of the type of that proposed by Bryan and Love is more nearly true. Most boiler flues used in modern boilers probably follow this formula. Arrangements are now being made to get results on large steel tubes such as are used in modern boiler construction, and it is hoped thus to eliminate the errors due to differences in annealing, etc., of small tubes, and test further the practical application of the formula.

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